BASIS

1. Find a basis for each of the following subspaces and state the dimension of each.

   (a) $S_1 = \text{Span}\Big\{(1,1,-3)\,,(2,2,-6)\,,(-4,-4,12)\Big\}$; (b) $S_2 = \Big\{(x,y,z) \in \Re^3 \,|\, y = 0\Big\}$

   (c) $S_3 = \Big\{(x,y,z) \in \Re^2 \,|\, x + 2y = 0\Big\}$; (d) $S_4 = \text{Span}\Big\{(1,1,2)\,,(-1,2,5)\,,(0,3,7)\,,(1,4,9)\Big\}$

   (e) $S_5 = \Big\{(x,y,z) \in \Re^3 \,|\, 2x + y - 3z = 0\Big\}$; (f) $S_6 = \text{Span}\Big\{(1,1,-3)\,,(2,2,-6)\,,(5,6,9)\Big\}$

2. None of the following is a basis for $\Re^3$. Why not?

   (a) $\Big\{(5,7,-2)\,,(-4,6,9)\Big\}$; (b) $\Big\{(0,0,0)\,,(1,2,3)\,,(4,5,6)\Big\}$

   (c) $\Big\{(3,5,-7)\,,(-2,4,1)\,,(5,1,-8)\Big\}$; (d) $\Big\{(1,1,0)\,,(0,1,1)\,,(1,0,1)\,,(1,1,1)\Big\}$

3. (a) What is the standard basis for $\Re^4$? ; (b) Find another basis for $\Re^4$.

4. Find a basis for the following subspaces. State the dimension of S .

   (a) $S = \Big\{(x,0) \in \Re^2\Big\}$; (b) $S = \Big\{(x,y) \in \Re^2 \,|\, x + 2y = 0\Big\}$; (c) $S = \Big\{(x,y,z) \in \Re^3 \,|\, z = 0\Big\}$

   (d) $S = \Big\{(-t,t,0) \;|\; \in \Re\Big\}$; (e) $S = \Big\{(x,y,z) \in \Re^3 \,|\, x - y - z = 0\Big\}$

   (f) $S = \Big\{(x,y,z) \in \Re^3 \,|\, 4x + y - z = 0\Big\}$; (g) $S = \Big\{(t,t,t) \;|\, t \in \Re\Big\}$

   (h) $S = \Big\{(x,y,z,w) \in \Re^4 \,|\, x + 3y = 0\,,\; z + w = 0\Big\}$

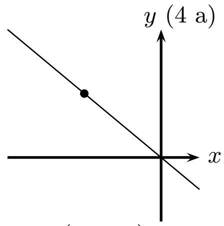
   (i) $S = \Big\{(x,y,z,w) \in \Re^4 \,|\, x + 2y - z + 5w = 0\Big\}$

   (j) $S = \Big\{(x,y,z,w) \in \Re^4 \,|\, x + y = z + w\Big\}$; (k) $S = \Big\{(a+b, a-b, a, b) \in \Re^4\Big\}$

   (l) $S = \Big\{(a, a+b, a-b, b) \in \Re^4\Big\}$; (m) $S = \Big\{(a-b, b+c, a, b+c) \in \Re^4\Big\}$

   (n) $S = \Big\{(0,0,0) \in \Re^3\Big\}$; (o) $\Re^3$; (p) $\Re^4$

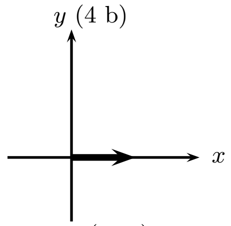
<u>BASIS</u>



$(-2,1)$

(4 a) basis: $\{(1,0)\}$; $d=1 \rightarrow$ this is a line in $\Re^2$ (the $x-$axis)

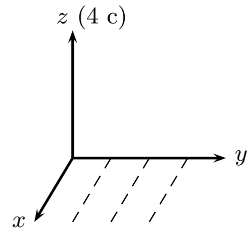
(4 b) basis: $\{(-2,1)\}$; $d=1 \rightarrow$ a line in $\Re^2$



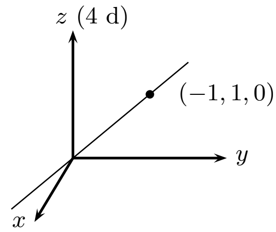
$(1,0)$

(4 c) basis: $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$; $d=2 \rightarrow$ a plane in $\Re^2 \rightarrow xy$ plane (floor)

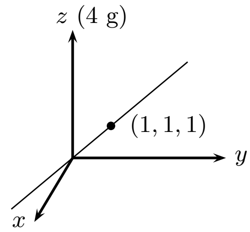
(4 d) basis: $\left\{ \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \right\}$; $d=1 \rightarrow$ this is a line in $\Re^3$



(4 e) basis: $\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}$; $d=2 \rightarrow$ a plane in $\Re^3$



(4 f) basis: $\left\{ \begin{pmatrix} -1/4 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1/4 \\ 0 \\ 1 \end{pmatrix} \right\}$; $d=2 \rightarrow$ a plane in $\Re^3$



(4 g) basis: $\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$; $d=1 \rightarrow$ line in $\Re^3$

(4 h) basis: $\left\{ \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \end{pmatrix} \right\}$; $d=2$; $\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} -3t \\ t \\ -s \\ s \end{pmatrix}$; a $2-$dimensional subspace of $\Re^4$

(4 i) basis: $\left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -5 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$; $d=3$; $\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} -2r+s-5t \\ r \\ s \\ t \end{pmatrix} = r \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -5 \\ 0 \\ 0 \\ 1 \end{pmatrix}$;

a $3-$dimensional subspace of $\Re^4$