

Linear Combinations

Given a set of vectors $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \dots, \vec{v}_n\}$; \vec{u} is a linear combination of the v 's if there exists $c_1, c_2, c_3, \dots, c_n$ such that $c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 + \dots + c_n\vec{v}_n = \vec{u}$

$\text{Span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \dots, \vec{v}_n\}$ = the set of all vectors which are Linear Combinations of the v 's .

If \vec{u} is a L.C. of the v 's then $\vec{u} \in \text{Span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \dots, \vec{v}_n\}$

(1) Given $\vec{u} = (1, 2, 3)$ and $\vec{v} = (4, 5, 6)$ or $\{(1, 2, 3), (4, 5, 6)\}$, determine whether the following vectors are L.C.'s of \vec{u} and \vec{v} .

(a) $(0, 0, 0)$; (b) $(1, 2, 3)$; (c) $(8, 10, 12)$; (d) $(7, 8, 9)$; (e) $(5, 7, 9)$
(f) $(2, -1, 5)$

(2) Given $\vec{u} = (4, 0, -2, -2)$, $\vec{v} = (1, -1, 2, 3)$, $\vec{w} = (0, 2, 1, 4)$

Determine whether each of the following vectors belongs to the $\text{Span}\{\vec{u}, \vec{v}, \vec{w}\}$.

(a) $(6, -8, -1, -8)$; (b) $(0, 0, 0, 0)$; (c) $(6, 0, 3, 8)$; (d) $(-1, 5, 7, 1)$

Linear Independence vs Linear Dependence

A set of vectors $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \dots, \vec{v}_n\}$ is a L.I. if $c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 + \dots + c_n\vec{v}_n = \vec{0}$

$\Rightarrow c_1 = c_2 = c_3 \dots c_n = 0$ (uniquely) . Otherwise, $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \dots, \vec{v}_n\}$ is L.D.

Determine whether the following sets of vectors are LI or LD.

(1) $\{\vec{u}, \vec{v}, \vec{w}\}$ where $4\vec{u} - 2\vec{v} + 3\vec{w} = \vec{0}$ (2) $\{\vec{u}, \vec{v}, \vec{w}\}$ where $0\vec{u} - 0\vec{v} - 3\vec{w} = \vec{0}$

(3) $\{\vec{u}, \vec{v}, \vec{w}\}$ where $0\vec{u} + 0\vec{v} + 0\vec{w} = \vec{0}$ and this is the only way of writing this equation

(4) $\{\vec{u}, \vec{v}, \vec{w}, \vec{0}\}$ (5) $\{(1, 2, 3), (2, 4, 6)\}$ (6) $\{(1, 2, 3), (4, 5, 6), (7, 8, 9)\}$

(7) $\{(1, 2, 3), (0, 2, 5), (1, 0, 4)\}$ (8) $\{(-2, 0, 1), (3, 2, 5), (6, -1, 1), (7, 0, -2)\}$

(9) $\{(1, 1, 0, -1), (0, 0, 2, 2), (3, 3, 0, 0)\}$

(10) $\{(3, 0, -3, 6), (0, 2, 3, 1), (0, -2, -2, 0), (-2, 1, 2, 1)\}$

Answers: LI : (3) , (7) , (9) , (10) ; LD : (1) , (2) , (4) , (5) , (6) , (8)