

Lines in \mathbb{R}^2 and \mathbb{R}^3

- (1) Find the equation of the line passing through (1 , -4) and (3 , 7)
 (a) in standard form ($Ax + By = C$) ; (b) in vector form ; (c) in parametric form.
- (2) Find the equation of the line passing through (1 , -5) with slope = -2/3
 (a) in standard form ; (b) in vector form ; (c) in parametric form
- (3) Find the equation of the line L which :
- (a) parallel to (2 , -1 , 0) and passing through $P(1, -1, 3)$
 (b) passing through $P(3, -1, 4)$ and $Q(1, 0, -1)$
 (c) passing through $P(3, -1, 4)$ and $Q(3, -1, 5)$
 (d) parallel to (1 , 2 , -7) and passing through (0 , 0 , 0)
 (e) passing through $P(1 , 0 , -3)$ and parallel to the line :
$$\begin{cases} x = -1 + 2t \\ y = 2 - t \\ z = 3 + 3t \end{cases}$$

 (f) passing through $P(2, -1, 1)$ and parallel to the line $(x, y, z) = (2, 1, 0) + t(-1, 0, 1)$

Answers:

(1) (a) $11x - 2y = 19$; (b) $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \end{pmatrix} + t \begin{pmatrix} 2 \\ 11 \end{pmatrix}$ or $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \end{pmatrix} + t \begin{pmatrix} 2 \\ 11 \end{pmatrix}$; (c) $\begin{cases} x = 1 + 2t \\ y = -4 + 11t \end{cases}$ or $\begin{cases} x = 3 + 2t \\ y = 7 + 11t \end{cases}$

(2) (a) $2x + 3y = -13$; (b) $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -5 \end{pmatrix} + t \begin{pmatrix} 3 \\ -2 \end{pmatrix}$; (c) $\begin{cases} x = 1 + 3t \\ y = -5 - 2t \end{cases}$; (3 a) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} + t \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$

(3 b) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} + t \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$ or $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + t \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$;

(3 c) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} + t \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix} + t \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$; (3 d) $(x, y, z) = t(1, 2, -7)$
 (3 e) $(x, y, z) = (1, 0, -3) + t(2, -1, 3)$
 (3 f) $(x, y, z) = (2, -1, 1) + t(-1, 0, 1)$

- (4) Verify that P and Q lie on the given line and R does not lie on the line.

(a) $l : (x, y, z) = (3, 2, 1) + t(-4, 1, -1)$
 $P(-1, 3, 0)$; $Q(11, 0, 3)$; $R(-1, 4, 5)$

(b) $l : (x, y, z) = (4, 3, 1) + t(-1, 0, -2)$
 $P(2, 3, -3)$; $Q(-1, 3, -9)$; $R(2, 3, 7)$