

Matrix Operations

(1) Find a, b, c, d if

$$(a) \begin{bmatrix} a-b & b-c \\ c-d & d-a \end{bmatrix} = 2 \begin{bmatrix} 1 & 1 \\ -3 & 1 \end{bmatrix}; (b) \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} b & c \\ d & a \end{bmatrix}; (c) \begin{bmatrix} a+b & 2c+d \\ a-b & 3c-a \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Answers: (a) $(a,b,c,d) = (-2+t, -4+t, -6+t, t)$; (b) $(a,b,c,d) = (t, t, t, t)$; i.e. $a=b=c=d$
(c) $(a,b,c,d) = (2, -1, 2, -2)$

(2)

$$(a) 3 \begin{bmatrix} 3 \\ -1 \end{bmatrix} - 5 \begin{bmatrix} 6 \\ 2 \end{bmatrix} + 7 \begin{bmatrix} 1 \\ -1 \end{bmatrix}; (b) \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -4 \\ -2 & 4 & 0 \end{bmatrix}^t; (c) 3 \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}^t - 2 \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

Answers: (a) $\begin{bmatrix} -14 \\ -20 \end{bmatrix}$; (b) $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 4 \\ 2 & -4 & 0 \end{bmatrix}$; (c) $\begin{bmatrix} 4 & -1 \\ -1 & -6 \end{bmatrix}$

(3) Find A if (a) $5A - \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} = 3A - \begin{bmatrix} 5 & 2 \\ 6 & 1 \end{bmatrix}$; (b) $3A + \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 5A - 2 \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

Answers: (a) $A = \begin{bmatrix} -2 & -1 \\ -2 & 1 \end{bmatrix}$; (b) $A = \begin{pmatrix} 4 \\ 1/2 \end{pmatrix}$

(4) Find A in terms of B if (a) $A + B = 3A + 2B$; (b) $2A - B = 5(A + 2B)$

Answers: (a) $A = -\frac{1}{2}B$; (b) $A = -\frac{11}{3}B$

(5) Simplify : Assume A, B and C are matrices of the same size.

(a) $2(9(A - B) + 7(2B - A)) - 2(3(2B + A) - 2(A + 3B) - 5(A + B))$

(b) $5[3(A - B + 2C) - 2(3C - B) - A] + 2[3(3A - B + C) + 2(B - 2A) - 2C]$

Answers: (a) $12A + 20B$; (b) $20A - 7B + 2C$

(6) Find s and t such that $A = A^t$

$$(a) A = \begin{pmatrix} 1 & s \\ -2 & t \end{pmatrix}; (b) A = \begin{pmatrix} s & t \\ st & 1 \end{pmatrix}; (c) A = \begin{pmatrix} s & 2s & st \\ t & -1 & s \\ t & s^2 & s \end{pmatrix}; (d) A = \begin{pmatrix} 2 & s & t \\ 2s & 0 & s+t \\ 3 & 3 & t \end{pmatrix}; (e) A = \begin{pmatrix} s & s^2 \\ st & t \end{pmatrix}$$

Answers :

(a) $s = -2, t = \text{any real number}$; (b) $s = 1, t = \text{any real number} \rightarrow \begin{bmatrix} 1 & t \\ t & 1 \end{bmatrix}$ or $t = 0, s = \text{any real number} \rightarrow \begin{bmatrix} s & 0 \\ 0 & 1 \end{bmatrix}$

(c) $s=0, t=0$ or $s=1, t=2$; (d) $s=0, t=3$; (e) $s=0, t=\text{any real number}$ or $s=t=\text{any real number}$

Matrix Operations

(7) Find A given :

$$(a) \left(A + 3 \begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & 4 \end{pmatrix} \right)^t = \begin{pmatrix} 2 & 1 \\ 0 & 5 \\ 3 & 8 \end{pmatrix}; \quad (b) \left(3A^t + 2 \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \right)^t = \begin{pmatrix} 8 & 0 \\ 3 & 1 \end{pmatrix}$$

$$(c) \left(2A - 3 \begin{pmatrix} 1 & 2 & 0 \end{pmatrix} \right)^t = 3A^t + \left(2 \ 1 \ -1 \right)^t$$

$$(d) \left(2A^t - 5 \begin{pmatrix} -1 & 0 \\ -1 & 2 \end{pmatrix} \right)^t = 4A - 9 \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\text{Answers: (b) } A = \begin{pmatrix} 2 & 0 \\ 1 & -1 \end{pmatrix}; \quad (c) A = (-5 \ -7 \ 1); \quad (d) A = \begin{pmatrix} 7 & 7 \\ -\frac{9}{2} & -5 \end{pmatrix}$$

(8) Find a, b, c, d if

$$(a) \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix}; \quad (b) \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 7 & 2 \\ -1 & 4 \end{pmatrix}$$

$$\text{Answer: (b) } \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 1 \\ 2 \end{pmatrix}$$

(9) Find all possible matrix products $A^2, AB, AC, BA, CA, BC, CB, C^2, B^2$ for

$$(a) A = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 0 & 0 \end{pmatrix}; \quad B = \begin{pmatrix} 1 & -2 \\ 1/2 & 3 \end{pmatrix}; \quad C = \begin{pmatrix} -1 & 0 \\ 2 & 5 \\ 0 & 3 \end{pmatrix}; \quad (b) A = \begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & -1 \end{pmatrix}; \quad B = \begin{pmatrix} -1 & 6 \\ 1 & 0 \end{pmatrix}; \quad C = \begin{pmatrix} 2 & 0 \\ -1 & 1 \\ 1 & 2 \end{pmatrix}$$

$$\text{Answers: (b) } BA = \begin{pmatrix} -1 & 4 & -10 \\ 1 & 2 & 4 \end{pmatrix}, \quad B^2 = \begin{pmatrix} 7 & -6 \\ -1 & 6 \end{pmatrix}, \quad CB = \begin{pmatrix} -2 & 12 \\ 2 & -6 \\ 1 & 6 \end{pmatrix}, \quad AC = \begin{pmatrix} 4 & 10 \\ -2 & -1 \end{pmatrix}, \quad CA = \begin{pmatrix} 2 & 4 & 8 \\ -1 & -1 & -5 \\ 1 & 4 & 2 \end{pmatrix}$$

(10) Verify that $A^2 - A - 6I = 0$ if (a) $A = \begin{pmatrix} 3 & -1 \\ 0 & -2 \end{pmatrix}$; (b) $A = \begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix}$

$$(11) \text{ Given } A = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}; \quad B = \begin{pmatrix} 1 & 0 & -2 \\ 3 & 1 & 0 \end{pmatrix}; \quad C = \begin{pmatrix} 1 & 0 \\ 2 & 1 \\ 5 & 8 \end{pmatrix}; \quad D = \begin{pmatrix} 3 & -1 & 2 \\ 1 & 0 & 5 \end{pmatrix}$$

Verify that (a) $A(BC) = (AB)C$; (b) $(CD)^t = D^t C^t$

Matrix Operations

(12) Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

(a) if $AB=BA$ where $B = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, show that $d=a, c=0$ so that $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ 0 & a \end{pmatrix}$

(b) if $AB=BA$ where $B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$, show that $d=a, b=0$ so that $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & 0 \\ c & a \end{pmatrix}$

(13) Find $A_{2 \times 2}$ such that : (a) $A^2 = 0$ and $A \neq 0$; (b) $A^2 = I$ and $A \neq I$

(14) Given $U = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ and $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $UA = 0$; prove $A = 0$ ($a = b = c = d = 0$)

(15) Simplify (assume A, B, C, D are square matrices) :

(a) $A(3B - C) + (A - 2B)C + 2B(C + 2A)$; (b) $A(B + C - D) + B(C - A + D) - (A + B)C + (A - B)D$
(c) $AB(BC - CB) + (CA - AB)BC + CA(A - B)C$; (d) $(A - B)(C - A) + (C - B)(A - C) + (C - A)^2$

Answers: (b) $AB - BA$; (d) 0

(16) Use $A_{2 \times 2}$, $B_{2 \times 2}$ (examples using numbers) to illustrate the following :

(a) $(A + B)^2 \neq A^2 + 2AB + B^2$; (b) $(A - B)(A + B) \neq A^2 - B^2$

(c) $AB = AC$ does not imply that $B = C$; (d) $AB = 0$ does not imply $A = 0$ or $B = 0$