

Rules for Operations

Matrix Multiplications

- (1) $A(BC) = A(BC)$ (association)
- (2) $(A + B)C = AC + BC$ (distributive)
 $A(B + C) = AB + AC$
- (3) $k(AB) = (kA)B = A(kB)$ (scalars are associative and commutative)
- (4) $A_{m \times n} I_n = A$; $I_m A_{m \times n} = A$; $IA = AI = I$ (if A is square)
- (5) $A_{m \times n} O_{n \times p} = O_{m \times p}$; $O_{q \times m} A_{m \times n} = O_{q \times n}$
 $AO = OA = O$ (if both A and O are square)
- (6) AB is a diagonal matrix if both A and B are diagonal.

Transposes

- (1) $(A^T)^T = A$
- (2) $(A + B)^T = A^T + B^T$
- (3) $(kA)^T = k(A^T)$
- (4) $(AB)^T = B^T A^T$
- (5) $(A^T)^3 = (A^T)(A^T)(A^T)$
 $= (A^3)^T$

Inverses (assume A^{-1}, B^{-1} both exist)

- (1) $(A^{-1})^{-1} = A$
- (2) $(A + B)^{-1} \neq A^{-1} + B^{-1}$
- (3) $(kA)^{-1} = \frac{1}{k}(A^{-1})$
- (4) $(AB)^{-1} = B^{-1}A^{-1}$
- (5) $(A^{-1})^3 = (A^{-1})(A^{-1})(A^{-1}) = (A^3)^{-1}$
 $= A^{-3}$
- (6) $(A^{-1})^T = (A^T)^{-1}$

Negative Statements (Prove these statements using 2×2 matrices with numerical entries)

- (1) $AB \neq BA$
- (2) $(A + B)^2 \neq A^2 + 2AB + B^2$
- (3) $(A - B)^2 \neq A^2 - 2AB + B^2$
- (4) $(A - B)(A + B) \neq A^2 - B^2$
- (5) $(A + B)^{-1} \neq A^{-1} + B^{-1}$ (in fact $(A + B)^{-1}$ may not even exist if both A^{-1} and B^{-1} exist)
- (6) $AB = O$ does not imply $A = O$ or $B = O$
- (7) $A^2 = O$ does not imply $A = O$
- (8) $A^2 = I$ does not imply $A = I$
- (9) $AB = AC$ does not imply $B = C$

Some theoretical results - using Inverses

- (1) $AB = AC$ and A^{-1} exists $\Rightarrow B = C$ (cancellation law)
- (2) $AB = O$ and A^{-1} exists $\Rightarrow B = O$
- (3) $A^2 = I$ and A^{-1} exists $\Rightarrow A = A^{-1}$

Square Matrices which commute (exceptions to the rule)

- (1) $AA^{-1} = A^{-1}A = I$ (a matrix and its inverse)
- (2) $AB = BA$ (if A and B are diagonal matrices)
- (3) $AI = IA = A$ (if A is square)
- (4) $AO = OA = O$ (if both A and O are square)

Square Matrices which do not have inverses

- (1) O
- (2) matrices with multiple rows (columns)
- (3) matrices with a row of zeros

In general, matrices whose determinant is zero are not invertible.