

## Subspaces or not ? Justify

A subset  $S$  of  $\mathbb{R}^n$  is a subspace of  $\mathbb{R}^n$  if  $S$  is closed under addition and closed under scalar multiplication.

$$(1) S = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \mid \begin{pmatrix} x \\ y \end{pmatrix} = t \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$$

$$(2) S = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \mid \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1-t \\ t \end{pmatrix} \right\}$$

$$(3) S = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \mid x + y = 0 \right\}$$

$$(4) S = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \mid x \geq 0 \right\}$$

$$(5) S = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1 \right\}$$

$$(6) S = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \mid x = t + 4, y = t, z = t - 2 \right\}$$

$$(7) S = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \mid xy = 0 \right\}$$

$$(8) S = \left\{ (x, y, z) \in \mathbb{R}^3 \mid x - y + z = 0 \right\}$$

$$(9) S = \left\{ (x, y, z) \in \mathbb{R}^3 \mid x = -2t, y = 5t, z = 3t \right\}$$

$$(10) S = \left\{ (x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 \leq 25 \right\}$$

$$(11) S = \left\{ (x, y, z) \in \mathbb{R}^3 \mid x + y = 0, z = 0 \right\}$$

$$(12) S = \left\{ (x, y, z) \in \mathbb{R}^3 \mid z = 0 \right\} \quad (13) S = \left\{ (x, y, z) \in \mathbb{R}^3 \mid 2x - 4y + 3z + 5 = 0 \right\}$$

$$(14) S = \left\{ (x, y, z) \in \mathbb{R}^3 \mid xy = z \right\} \quad (15) S = \left\{ (x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 0 \right\}$$

### Answers

$$(1) t = 0: \begin{pmatrix} 0 \\ 0 \end{pmatrix} \in S ; \text{ Let } \vec{u} = a \begin{pmatrix} 1 \\ 2 \end{pmatrix}; \vec{v} = b \begin{pmatrix} 1 \\ 2 \end{pmatrix} \in S ; \vec{u} + \vec{v} = (a + b) \begin{pmatrix} 1 \\ 2 \end{pmatrix} \in S ;$$

$$k\vec{u} = (ka) \begin{pmatrix} 1 \\ 2 \end{pmatrix} \in S ; \text{ both closure hold , therefore } S \text{ is a subspace of } \mathbb{R}^2$$

$$(2) \begin{pmatrix} 0 \\ 0 \end{pmatrix} \notin S \text{ ( no } t \text{ value to get } \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ ) ; not a subspace of } \mathbb{R}^2$$

$$\text{closure under addition fails: } \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \in S ; \text{ but } \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \notin S \left( \text{ no } t \text{ value to get } \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right)$$

$$\text{closure under scalar multiplication fails: } \begin{pmatrix} 1 \\ 0 \end{pmatrix} \in S ; \text{ but } 5 \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \notin S \left( \text{ no } t \text{ value to get } \begin{pmatrix} 5 \\ 0 \end{pmatrix} \right)$$

## Subspaces or not? Justify

### Answers

(3) elements of form  $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -t \\ t \end{pmatrix} = t \begin{pmatrix} -1 \\ 1 \end{pmatrix} \in S$ ; Let  $\vec{u} = a \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ ;  $\vec{v} = b \begin{pmatrix} -1 \\ 1 \end{pmatrix} \in S$ ;  $\vec{u} + \vec{v} = (a+b) \begin{pmatrix} -1 \\ 1 \end{pmatrix} \in S$ ;

$k\vec{u} = (ka) \begin{pmatrix} -1 \\ 1 \end{pmatrix} \in S$ ; both closure hold, therefore  $S$  is a subspace of  $\mathbb{R}^2$

(4) not a subspace of  $\mathbb{R}^2$ :  $\begin{pmatrix} 0 \\ 0 \end{pmatrix} \in S$ ; closure under addition holds:  $a, c \geq 0 \rightarrow \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a+c \\ b+d \end{pmatrix} \in S$  since  $a+c \geq 0$

closure under scalar multiplication fails:  $\begin{pmatrix} 5 \\ 6 \end{pmatrix} \in S$ ; but  $(-1) \begin{pmatrix} 5 \\ 6 \end{pmatrix} = \begin{pmatrix} -5 \\ -6 \end{pmatrix} \notin S$

(5)  $\begin{pmatrix} 0 \\ 0 \end{pmatrix} \in S$ ; Let  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ;  $\begin{pmatrix} 0 \\ 1 \end{pmatrix} \in S$ ; but  $\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \notin S$  since  $1^2 + 1^2 = 2 > 1$ ;  $5 \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \notin S$  since  $5^2 + 0^2 = 25 > 1$ ;

both closure fail, therefore  $S$  is not a subspace of  $\mathbb{R}^2$

(6)  $\begin{pmatrix} 0 \\ 0 \end{pmatrix} \notin S$ ; Let  $t=0 \rightarrow \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix}$ ;  $t=1 \rightarrow \begin{pmatrix} 5 \\ 1 \\ -1 \end{pmatrix} \in S$ ; but  $\begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix} + \begin{pmatrix} 5 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 9 \\ 1 \\ -3 \end{pmatrix} \notin S$ ;  $5 \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} 20 \\ 0 \\ -10 \end{pmatrix} \notin S$  since  $\begin{cases} t+4=20 \Rightarrow t=16 \\ t=0 \end{cases}$ ;

since  $\begin{cases} t+4=9 \Rightarrow t=5 \\ t=1 \end{cases}$  contradictory  $t$ 's

both closure fail, therefore  $S$  is not a subspace of  $\mathbb{R}^2$

(7)  $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \in S$ ; Let  $\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ ;  $\begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} \in S$ ; but  $\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix} \notin S$  since  $(1)(1) \neq 0$ ;  $5 \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 10 \end{pmatrix} \in S$ ;

closure under addition fails, therefore  $S$  is not a subspace of  $\mathbb{R}^2$

Note: scalar multiplication works;  $(x, y, z) \in S \Rightarrow xy = 0$ ,  $k(x, y, z) \in S$  since

$$(kx)(ky) = k^2(xy) = k^2(0) = 0.$$

(8) parametric form  $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} s-t \\ s \\ t \end{pmatrix} = s \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ ;  $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \in S$ ; Let  $\vec{u} = a \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + b \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ ;  $\vec{v} = c \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + d \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \in S$ ;

$\vec{u} + \vec{v} = (a+c) \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + (b+d) \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \in S$ ;  $k\vec{u} = (ka) \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + (kb) \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \in S$ ; both closure hold, then  $S$  is a subspace of  $\mathbb{R}^3$

(9)  $t=0 \rightarrow \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \in S$ ; Let  $\vec{u} = a \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix}$ ;  $\vec{v} = b \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix} \in S$ ;  $\vec{u} + \vec{v} = (a+b) \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix} \in S$ ;  $k\vec{u} = (ka) \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix} \in S$ ;

both closure hold, therefore  $S$  is a subspace of  $\mathbb{R}^3$

(10)  $(0, 0, 0) \in S$ ; Let  $\vec{u} = (0, 0, 5)$ ;  $\vec{v} = (0, 5, 0) \in S$ ;  $\vec{u} + \vec{v} = (0, 5, 5) \notin S$  since  $5^2 + 5^2 = 50 > 25$ ;

$4\vec{u} = (0, 0, 20) \notin S$  since  $20^2 = 400 > 25$ ; both closure fail, therefore  $S$  is not a subspace of  $\mathbb{R}^3$

## Subspaces or not ? Justify

### Answers

$$(11) \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -t \\ t \\ 0 \end{pmatrix} = t \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \in S ; \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \in S ; \text{Let } \vec{u} = a \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} ; \vec{v} = b \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \in S ; \vec{u} + \vec{v} = (a+b) \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \in S ; k\vec{u} = (ka) \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \in S ;$$

both closure hold , therefore  $S$  is a subspace of  $\mathbb{R}^3$

$$(12) \text{ parametric form } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} s \\ t \\ 0 \end{pmatrix} = s \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} ; \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \in S ; \text{Let } \vec{u} = a \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + b \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} ; \vec{v} = c \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + d \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \in S ;$$

$$\vec{u} + \vec{v} = (a+c) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + (b+d) \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \in S ; k\vec{u} = (ka) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + (kb) \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \in S ; \text{both closure hold , then } S \text{ is a subspace of } \mathbb{R}^3$$

$$(13) (0,0,0) \notin S ; \text{Let } \vec{u} = (0,0,-\frac{5}{3}) ; \vec{v} = (-\frac{5}{2},0,0) \in S ; \vec{u} + \vec{v} = (-\frac{5}{2},0,-\frac{5}{3}) \notin S \text{ since } 2(-\frac{5}{2}) - 4(0) + 3(-\frac{5}{3}) + 5 \neq 0 ;$$

$$3\vec{u} = (0,0,-5) \notin S \text{ since } 2(0) - 4(0) + 3(-5) + 5 \neq 0 ;$$

both closure fail , therefore  $S$  is not a subspace of  $\mathbb{R}^3$

$$(14) (0,0,0) \in S ; \text{Let } (1,5,5) ; (3,4,12) \in S ; (1,5,5) + (3,4,12) = (4,9,17) \notin S \text{ since } (4)(9) \neq 17 ;$$

$$3(1,5,5) = (3,15,15) \notin S \text{ since } (3)(15) \neq 15 ;$$

both closure fail , therefore  $S$  is not a subspace of  $\mathbb{R}^3$

$$(15) \text{ this is just the } \vec{0} = (0,0,0) \in S ; (0,0,0) + (0,0,0) = (0,0,0) \in S ; k\vec{0} = \vec{0} \in S ;$$

both closure hold , therefore  $S$  is a subspace of  $\mathbb{R}^3$