

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \dots \quad \text{infinite series}$$

$$S_1 = a_1 ; S_2 = a_1 + a_2 ; S_3 = a_1 + a_2 + a_3 ; \dots$$

$$S_n = a_1 + a_2 + a_3 + \dots + a_n$$

$$\{ S_1 , S_2 , S_3 , \dots , S_n , \dots \} = \text{sequence of partial sums}$$

GEOMETRIC SERIES

$$a + ar + ar^2 + ar^3 + \dots$$

$$S_1 = a ; S_3 = a + ar + ar^2 ; \dots \rightarrow S_n = a + ar + ar^2 + \dots + ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (\text{in compressed form})$$

$$S = \text{sum of the series} = \lim_{n \rightarrow \infty} S_n = S_{\infty} = \frac{a}{1-r} \quad (\text{assuming this limit exists})$$

Otherwise, the series diverges ( that is, the series has no sum ).

Geometric Series

Find  $S_n$  and determine whether the series converges. If the series converges, find the sum of the series.

$$(1) \quad \sum_{k=1}^{\infty} \frac{2}{5^{k-1}} \quad S_n = \frac{5}{2} \left[ 1 - \left( \frac{1}{5} \right)^n \right] ; \text{convergent to } S_{\infty} = \frac{5}{2}$$

$$(2) \quad \sum_{k=1}^{\infty} \frac{2^{k-1}}{4} \quad S_n = -\frac{1}{4} [ 1 - (2)^n ] ; \text{divergent}$$

$$(3) \quad \sum_{n=1}^{\infty} 4^{n-1} \quad S_n = -\frac{1}{3} [ 1 - (4)^n ] ; \text{divergent}$$

$$(4) \quad \sum_{n=1}^{\infty} \left( -\frac{3}{2} \right)^{n+1} \quad S_n = \frac{9}{10} \left[ 1 - \left( -\frac{3}{2} \right)^n \right] ; \text{divergent}$$

$$(5) \quad \sum_{n=1}^{\infty} \frac{4^{n+2}}{7^{n+1}} \quad S_n = \frac{64}{21} \left[ 1 - \left( \frac{4}{7} \right)^n \right] ; \text{convergent to } S_{\infty} = \frac{64}{21}$$

$$(6) \sum_{k=1}^{\infty} \left( \frac{e}{\pi} \right)^{k-1} \quad S_n = \frac{\pi}{\pi-e} \left[ 1 - \left( \frac{e}{\pi} \right)^n \right] ; \text{ convergent to } S_{\infty} = \frac{\pi}{\pi-e}$$

$$(7) \sum_{k=1}^{\infty} \left( -\frac{1}{2} \right)^k \quad S_n = -\frac{1}{3} \left[ 1 - \left( -\frac{1}{2} \right)^n \right] ; \text{ convergent to } S_{\infty} = -\frac{1}{3}$$

$$(8) \sum_{n=1}^{\infty} \left( \frac{2}{3} \right)^{n+1} \quad S_n = \frac{4}{3} \left[ 1 - \left( \frac{2}{3} \right)^n \right] ; \text{ convergent to } S_{\infty} = \frac{4}{3}$$

$$(9) \sum_{k=1}^{\infty} \left( -\frac{3}{4} \right)^{k-1} \quad S_n = \frac{4}{7} \left[ 1 - \left( -\frac{3}{4} \right)^n \right] ; \text{ convergent to } S_{\infty} = \frac{4}{7}$$

$$(10) \sum_{k=1}^{\infty} (-1)^{k-1} \frac{7}{6^{k-1}} \quad S_n = 6 \left[ 1 - \left( -\frac{1}{6} \right)^n \right] ; \text{ convergent to } S_{\infty} = 6$$

### TELESCOPING SERIES

Find  $S_n$  and if possible, find the sum of the series

$$(1) \sum_{n=1}^{\infty} \frac{1}{n(n+1)} \quad S_n = 1 - \frac{1}{n+1} ; \text{ convergent to } S_{\infty} = 1$$

$$(2) \sum_{n=1}^{\infty} \frac{1}{n(n+2)} \quad S_n = \frac{1}{2} \left[ 1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2} \right] ; \text{ convergent to } S_{\infty} = \frac{3}{4}$$

$$(3) \sum_{k=1}^{\infty} \frac{6}{(3k+2)(3k-1)} \quad S_n = 1 - \frac{2}{3n+2} ; \text{ convergent to } S_{\infty} = 1$$

$$(4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)} \quad S_n = \frac{1}{2} \left[ 1 - \frac{1}{2n+1} \right] ; \text{ convergent to } S_{\infty} = \frac{1}{2}$$

$$(5) \sum_{n=1}^{\infty} [ \sin(n) - \sin(n+1) ]$$

$S_n = \sin(1) - \sin(n+1) ; \text{ divergent ( oscillation ) , no sum}$

$$(6) \sum_{n=1}^{\infty} \ln \left( \frac{n}{n+1} \right) \quad S_n = \ln(1) - \ln(n+1) = -\ln(n+1) ; \text{ divergent , no sum}$$

$$(7) \sum_{k=1}^{\infty} \left( \frac{1}{k+3} - \frac{1}{k+4} \right) \quad S_n = \frac{1}{4} - \frac{1}{n+4} ; \text{ convergent to } S_{\infty} = \frac{1}{4}$$

$$(8) \sum_{k=1}^{\infty} \frac{1}{(k+1)(k+2)} \quad S_n = \frac{1}{2} - \frac{1}{n+2} ; \text{ convergent to } S_{\infty} = \frac{1}{2}$$

$$(9) \sum_{n=1}^{\infty} \frac{4}{(n+3)(n+5)} \quad S_n = \frac{2}{4} + \frac{2}{5} - \frac{2}{n+4} - \frac{2}{n+5} ; \text{ convergent to } S_{\infty} = \frac{9}{10}$$

Express the repeating decimal as a fraction by writing the decimal as a geometric series.

$$(1) \mathbf{0.\overline{4}} \quad \text{converges to } \frac{4}{9} ; (2) \mathbf{0.\overline{9}} \quad \text{converges to } 1$$

$$(3) \mathbf{5.\overline{37}} \quad \text{converges to } \frac{532}{99} ; (4) \mathbf{0.\overline{159}} \quad \text{converges to } \frac{159}{999}$$

$$(5) \mathbf{0.\overline{7821}} \quad \text{converges to } \frac{79}{101} ; (6) \mathbf{0.451\overline{14}} \quad \text{converges to } \frac{44663}{99000}$$