(Marks)

1. Evaluate each of the following integrals without the use of integration tables.

(3) (a) $\displaystyle\int \frac{3\sqrt{t} - e + t\cos(2t)}{2t}\,dt$

(3) (b) $\displaystyle\int (x^3+2)\sin(x^4+8x)\,dx$

(3) (c) $\displaystyle\int \frac{2x}{(2x+1)^{3/2}}\,dx$

(4) (d) $\displaystyle\int_1^e \frac{\ln x}{x^2}\,dx$

(4) (e) $\displaystyle\int x^2 e^{2x}\,dx$

(4) (f) $\displaystyle\int \frac{3x^2+3x+1}{x(x+1)^2}\,dx$

(4) (g) $\displaystyle\int_0^{\sqrt{\frac{\pi}{4}}} x\sec^2(x^2)\,dx$

(4) 2. Given $f''(x) = 8e^{2x} - 3\sin x + 2$ with $f'(0) = 10$ and $f(0) = 1$, find $f(x)$.

3. Use the table of integrals to solve each of the following. In each case, state the formula number and justify its use.

(4) (a) $\displaystyle\int \frac{1}{\sqrt{x^2+10x+22}}\,dx$

(4) (b) $\displaystyle\int \frac{e^{2x}}{\sqrt{5+4e^x}}\,dx$

(8) 4. Solve the differential equation $\sec x\,\dfrac{dy}{dx} = \dfrac{1}{3y^2}$ with condition $y(\frac{\pi}{2}) = 2$.

(6) 5. Assume that the common cold virus spreads through the college student community at a rate proportional to the number of those infected. Five students were infected on the day of the outbreak (day 0), and by the following day (day 1), 40 had contracted the cold virus.

(a) Write the differential equation using the variables N and t, where N denotes the number of infected students and t denotes the number of days since the beginning of the outbreak.

(b) Solve the differential equation for N as a function of t.

(c) How many students will be infected on the second day after the outbreak (day 2)?

(4) 6. Given the curves $f(x) = x^2 - 10$ and $g(x) = 2x - 10$, determine

(a) the point(s) of intersection of $f(x)$ and $g(x)$,

(b) the area bounded by $f(x)$ and $g(x)$.

(4) 7. Use the trapezoidal rule with $n = 6$ to approximate $\displaystyle\int_0^3 \frac{1}{x^3+1}\,dx$. Round your answer to four decimal places.

(5) 8. Given the demand function $p = -x^2 + 100$ and the supply function $p = 15x$,

(a) find the equilibrium point,

(b) sketch and identify the regions representing the consumer and producer surpluses,

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(c) evaluate the consumer surplus.

(6) 9. Use l'Hôpital's rule to evaluate the following limits

(a) $\lim_{x\to 0} \dfrac{5x - \sin(2x)}{1 - e^x}$

(b) $\lim_{x\to\infty} \dfrac{\ln(2 + e^x)}{4x}$

(8) 10. Evaluate each improper integral and state whether it converges or diverges

(a) $\int_{-\infty}^{1} x^2 e^{x^3}\, dx$

(b) $\int_{0}^{3} \dfrac{x}{x^2 - 9}\, dx$

(6) 11. Determine the convergence or divergence of each sequence $\{a_n\}$. If the sequence converges, find the limit.

(a) $a_n = \left(\dfrac{7}{5}\right)^n$

(b) $a_n = \dfrac{3(n-1)!}{n!}$

(4) 12. Consider the sequence $\dfrac{5}{1}, \dfrac{2}{3}, \dfrac{-1}{9}, \dfrac{-4}{27}, \dfrac{-7}{81}, \ldots$

(a) Give the next three terms of the sequence.

(b) Give an expression for the n^{th} term of the sequence.

(3) 13. Given a repeated decimal $8.\overline{5}$, express it using a geometric series, find the sum of the geometric series and write the decimal as the ratio of two integers.

(6) 14. Determine with *justification* if each of the following series is convergent or divergent. If the series is convergent, *find its sum*.

(a) $\sum_{k=2}^{\infty} \dfrac{2^{k+1}}{3^{k-1}}$

(b) $\sum_{n=1}^{\infty} \dfrac{\sqrt{9n^2+1}}{5n+2}$

(3) 15. An amount of \$1200 is deposited in a bank that pays 2.4% interest per year compounded monthly. Find the balance at the end of two years.

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Answers

1.(a) $3\sqrt{t} - \frac{e}{2}\ln|t| + \frac{1}{4}\sin(2t) + C$ (b) $-\frac{1}{4}\cos(x^4 + 8x) + C$ (c) $\sqrt{2x+1} + \frac{1}{\sqrt{2x+1}} + C$

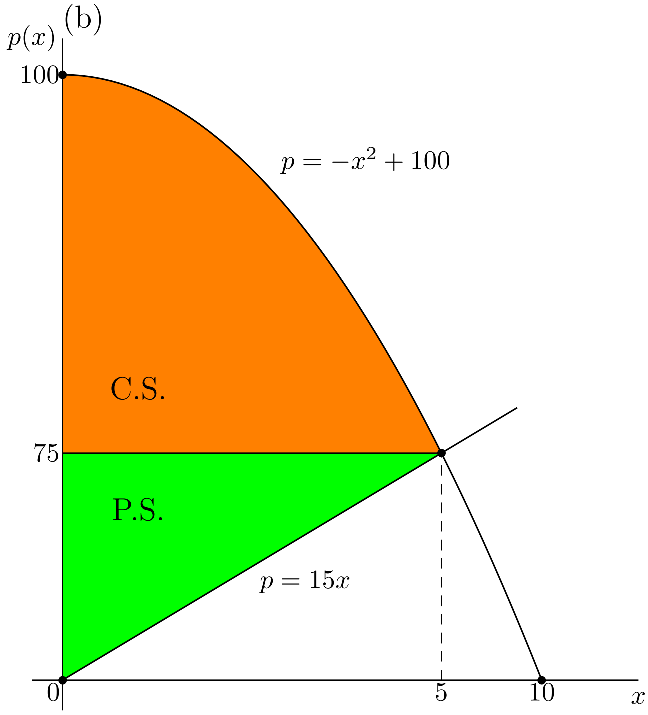
(d) $1 - \frac{2}{e}$ (e) $\frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$ (f) $\ln|x| + 2\ln|x+1| + \frac{1}{x+1} + D$ (g) $\frac{1}{2}$

2. $f(x) = 2e^{2x} + 3\sin x + x^2 + 3x - 1$ 3.(a) $\ln\left|x + 5 + \sqrt{(x+5)^2 - 3}\right| + C$

(b) $\frac{1}{24}(4e^x - 10)\sqrt{5 + 4e^x} + C$ 4. $y = \sqrt[3]{\sin x + 7}$ 5.(a) $\frac{dN}{dt} = kN$ (b) $N = e^{kt+C}$ (c) 320 students

6.(a) (0,-10) and (2,-6) (b) Area$= \frac{4}{3}$ 7. Area≈ 1.1533 8.(a) $(x_e, p_e) = (5, 75)$

(b)



(c) C.S.=\$83.33

9.(a) -3 (b) $\frac{1}{4}$ 10.(a) converges to $\frac{e}{3}$ (b) diverges to -∞ 11.(a) diverges (b) converges to 0

12.(a) $\frac{-10}{243}$, $\frac{-13}{729}$, $\frac{-16}{2187}$ (b) $a_n = \frac{8 - 3n}{3^{n-1}}$ 13. $\frac{77}{9}$ 14.(a) converges to 8 (b) diverges

15. \$1258.94