Final Exam 301RE – Winter 2009 - Answers

1a) ${}_4C_1 \cdot {}_{48}C_3 = 69184$

1b) ${}_4C_1 \cdot {}_{48}C_3 = 69184$

1c) $1 \cdot {}_{39}C_3 \; + \; 3 \cdot {}_{38}C_3 = 9139 + 25308 = 34447$

2a) ${}_{25}P_3$; 2b) ${}_{25}C_3$; 2c) $25 \cdot {}_{24}C_2$

3a) $\frac{4}{10} = 0.4$; 3b) $\frac{6}{10} = 0.6$

4a) $\frac{61}{417}$; 4b) $\frac{65}{115}$; 4c) $\frac{65}{180}$

5a) $0.03 + 0.11 + 0.28 = 0.42$; 5b) 0.07

6a) $P(x < 6) = P\left(Z < \frac{6 - 6.4}{0.3}\right) = P(Z < -1.33).....0.0918$

6b) $P(x > 7) = P\left(Z > \frac{7 - 6.4}{0.3}\right) = P(Z > 2).....0.0228$

6c) $P(6 < x < 7) = P\left(\frac{6 - 6.4}{0.3} < Z < \frac{7 - 6.4}{0.3}\right)$

$P(-1.33 < Z < 2) = 0.9772 - 0.0918 = 0.8854$

7a)

$P(\bar{x} < 10) \qquad Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \qquad \mu_{\bar{x}} = \mu = 12 \qquad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3.7}{\sqrt{20}}$

$Z = \frac{10 - 12}{\frac{3.7}{\sqrt{20}}} = -2.42 \qquad P(Z < -2.42) = 0.0078$

7b)

$P(10 < \bar{x} < 13).....P\left(\frac{10 - 12}{\frac{3.7}{\sqrt{20}}} < Z < \frac{13 - 12}{\frac{3.7}{\sqrt{20}}}\right)....P(-2.42 < Z < 1.21)$

$0.8869 - 0.0078 = 0.8791$

8)

$n = 30 \quad p = 0.60$

$np = 30(0.6) = 18 > 5$

$nq = 30(0.40) = 12 > 5$

$\mu_{\hat{p}} = p = 0.6 \quad \sigma_{\hat{p}} = \sqrt{\frac{(0.6)(0.4)}{30}} = 0.089$

$\frac{0.5}{n} = \frac{0.5}{30} = 0.017$

$P(\hat{p} < 0.5) = P(x < 0.5 - 0.17) = P(x < 0.483)$

$= P\left(Z < \frac{0.483 - 0.6}{0.089}\right) = P(Z < -1.31) = 0.0951$

interval is 89.04 to 97.36

9)

$n = 110 \quad \bar{x} = 93.2 \quad \sigma = 16.9 \quad Z_{0.99} = 2.58$

$\bar{x} - E = 93.2 - 4.16 = 89.04$

$\bar{x} + E = 93.2 + 4.16 = 97.36$

10)

$df = 10 - 1 = 9$

$t_{0.95} = 2.262$

$E = t_{0.95}\frac{s}{\sqrt{n}} = 2.262\frac{5.31}{\sqrt{10}} = 3.798 = 3.8$

$14.7 - 3.8 = 10.9$

$14.7 + 3.8 = 18.5$

$10.9 \ to \ 18.5$

11)

$\alpha = 0.01$

$H_0 : \mu = 5.3$

$H_1 : \mu < 5.3$

Left Tailed standard normal σ is known

$\bar{x} \quad Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{4.2 - 5.3}{\frac{1.9}{\sqrt{32}}} = -3.275 = -3.28$

$Z < -3.28 \qquad p - value = 0.0005$

$p - value \ 0.0005 \ < \ \alpha \ 0.01$

$\therefore \ we \ reject \ H_0$

12)

$\alpha = 0.01$

$H_0 : \ p = 0.7$

$H_1 : p \neq 0.70$

$use \ s\tan dard \ \ normal \ distribution$

$np = 32(0.7) = 22.4 > 5$

$nq = 32(0.3) = 9.6 > 5$

$\hat{p} = \frac{r}{n} = \frac{24}{32} = 0.75$

$Z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.75 - 0.7}{\sqrt{\frac{0.70(0.30)}{32}}} = 0.62$

$p - value = 2P(Z > 0.62) = 2(0.2676) = 0.5352$

$p - value \ 0.5352 \ > \ \alpha - value \ 0.01$

$we \ \ fail \ to \ reject \ H_0$

13)

$\alpha = 0.01$

$H_0 : \mu_d = 0$

$H_1 : \mu_d > 0$

right tailed test

Company	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Before Program (1) - After Program(2)	5	2	1	-5	0	5	11

$\overline{d} = \frac{29}{7} = 4.14 \qquad S_d = \sqrt{\frac{\sum (d - \overline{d})^2}{n - 1`}}$

$\frac{(5 - 4.14)^2}{6} + \frac{(2 - 4.14)^2}{6} + \frac{(1 - 4.14)^2}{6} + \frac{(-5 - 4.14)^2}{6} + \frac{(0 - 4.14)^2}{6} +$

$\frac{(5 - 4.14)^2}{6} + \frac{(11 - 4.14)^2}{6} = \sqrt{24.9} = 4.99$

$t = \frac{\overline{d}\sqrt{n}}{s\overline{d}} = \frac{(2.71)\sqrt{7}}{4.99} = 1.440$ .... $P - value$: 0.100

Conclusion $0.100 > \alpha = 0.05$ we do not reject H_0

14)

$\alpha = 0.05$

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 > \mu_2$

$Z = \frac{(\overline{x_1} - \overline{x_2}) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(53 - 47) - (0)}{\sqrt{\frac{7.3^2}{46} + \frac{9.1^2}{40}}} = 3.34$

$Z > 3.34$

$p - value$.....$1 - 0.9992 = 0.0008$.....0.0016

$p - value$ $0.0016 < \alpha = 0.05$

we reject H_0

15)

$\alpha = 0.01$

H_0 : *The distribution of fish has not changed*

H_1 : *The distribution of fish has changed*

$df = 4 - 1 = 3$

$\chi^2 = 11.64 + 1.36 + 0.27 + 4.91$

$\chi^2 = 18.18$

$p - value < 0.005$

$p - value < \alpha$

we reject H_0

16)

$P(U) = 0.12$ $\qquad$ $P(+/U) = 0.75$

$P(U') = 0.88$ $\qquad$ $P(+/U') = 0.07$

$$P(U/+) = \frac{P(U) \cdot P(+/U)}{P(U)P(+/U) + P(U')P(+/U')}$$

$$= \frac{(0.12)(0.95)}{(0.12)(0.95) + (0.88)(0.07)} = \frac{0.114}{0.1756} = 0.6492$$

17)

mean : $\quad np = (5)(0.75) = 3.75$

standard deviation $= \sqrt{npq} = \sqrt{(5)(0.75)(0.25)} = 0.968$