

(Marks)

- (12) 1. Use algebraic techniques to evaluate the following limits. Identify the limits that do not exist and use $-\infty$ or ∞ as appropriate. Show your work.

(a) $\lim_{x \rightarrow -3} \frac{x^2 - 2x - 15}{x + 3}$

(b) $\lim_{x \rightarrow -2} \frac{\sqrt{x+3} - 1}{x + 2}$

(c) $\lim_{x \rightarrow -\infty} \frac{2x - 6}{x + 1}$

(d) $\lim_{x \rightarrow \infty} \left(3 + \frac{5}{\sqrt{x}} \right)$

(e) $\lim_{x \rightarrow 2^-} f(x)$, where $f(x) = \begin{cases} 3x - 1 & \text{if } x < 2 \\ x^2 + 4 & \text{if } x \geq 2 \end{cases}$

(f) $\lim_{x \rightarrow -3^+} \frac{x + 2}{x^2 + 6x + 9}$

- (4) 2. Use the graph of the function $f(x)$ below to find the following. Use ∞ , $-\infty$, or DNE where appropriate.

(a) $\lim_{x \rightarrow -\infty} f(x) = \text{-----}$

(b) $\lim_{x \rightarrow -3^-} f(x) = \text{-----}$

(c) $\lim_{x \rightarrow -3^+} f(x) = \text{-----}$

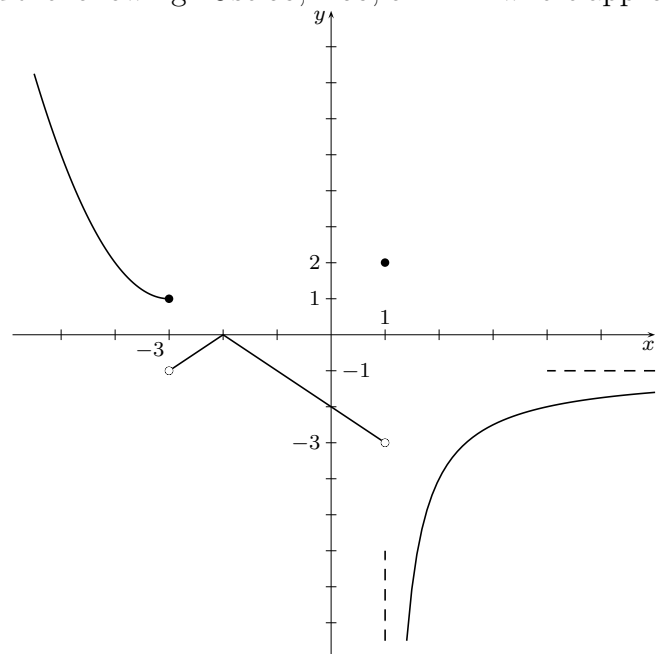
(d) $\lim_{x \rightarrow 1^-} f(x) = \text{-----}$

(e) $\lim_{x \rightarrow 1^+} f(x) = \text{-----}$

(f) $\lim_{x \rightarrow \infty} f(x) = \text{-----}$

(g) $f(-2) = \text{-----}$

(h) $f(1) = \text{-----}$



- (3) 3. For the function $f(x)$ as defined in question number 2,

(a) list the value(s) of x where $f(x)$ is discontinuous,

(b) list the value(s) of x where $f(x)$ is continuous but not differentiable.

- (2) 4. Find the value(s) of x for which the following function is not continuous. Justify using the definition of continuity.

$$f(x) = \begin{cases} 24 - x^2 & \text{if } x \leq 5 \\ |4 - x| & \text{if } x > 5 \end{cases}$$

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- (2) 5. Find the value(s) of k that will make $f(x)$ continuous for all real numbers.

$$f(x) = \begin{cases} \frac{x^2 + 2x - 3}{x - 1} & \text{if } x \neq 1 \\ k^2 & \text{if } x = 1 \end{cases}$$

- (8) 6. Given $f(x) = \sqrt{5 - x}$,
- (a) find $f'(x)$ **using the limit definition of the derivative**,
- (b) find the equation of the tangent line to $f(x)$ at $x = 1$.

- (24) 7. Find $\frac{dy}{dx}$ for each of the following functions. **Do not simplify your answers.**

(a) $y = 5x\sqrt{x^2 + 1}$

(b) $y = \frac{6x^2}{x^{1/2}} + \frac{5}{2x} - \frac{e}{\sqrt{x}}$

(c) $y = \frac{\sin(3x)}{3 - 2\cos x}$

(d) $y = \tan(2x) + e^{\sec 3x}$

(e) $y = \ln\left(\frac{\cos x}{\sqrt[3]{x^3 + 2}(3x - 1)^2}\right)$

(f) $y = \log_7(x^4 - 3x) + 2^{3x}$

(g) $e^y = x^2y^3 + 4$

(h) $y = (3x^2 + 5)^{\sqrt{x}}$

- (3) 8. Given the function $f(x) = x \sin(2x)$, find $f''(0)$.

- (4) 9. Find the x -values of the points on the graph of $f(x) = \frac{2}{3}x^3 - \frac{1}{2}x^2 - x$ where the slope of the tangent line is 2.

- (4) 10. Find the absolute extrema of $f(x) = 2x^3 - 9x^2 + 3$ on the interval $[2, 5]$.

(10) 11. Given $f(x) = \frac{2x^2}{(x-1)^2}$ $f'(x) = \frac{-4x}{(x-1)^3}$ $f''(x) = \frac{4(2x+1)}{(x-1)^4}$

- (a) find all x and y intercepts, vertical and horizontal asymptotes, intervals where $f(x)$ is increasing or decreasing, relative extrema, intervals where $f(x)$ is concave up or concave down, and points of inflection.

- (b) sketch the graph of $f(x)$ on the following page.

- (4) 12. Given $f(x) = 3x^4 - 18x^2 + 24x + 5$ $f'(x) = 12(x-1)^2(x+2)$ $f''(x) = 36x^2 - 36$
find all relative extrema of $f(x)$.

- (5) 13. A car dealer sells 80 cars per month at a price of \$20 000 per car. For every decrease of \$1000 in the price, 10 more cars are sold. What is the price to maximize the revenue?
(Be sure to use a test to confirm that this is a maximum.)

- (4) 14. The average cost function in \$/unit is given by $\bar{C} = -2x^2 + 3x + \frac{10}{x}$ where x is the number of units produced. Find the marginal cost at $x = 50$ and interpret the result.

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- (5) 15. A company wants to build a rectangular fence next to the wall of a building. No fencing is required along the wall of the building. The fence costs \$40 per meter. The company has \$4000 to spend for the fence. Find the dimensions of the courtyard that will maximize the area.
(Use a test to confirm that this is a maximum.)

(6) 16. Given the demand function $p = \frac{2}{\sqrt{7+x^2}}$,

- (a) find the price elasticity of demand function (simplify your answer),
 (b) find the price elasticity of demand at $x = 3$,
 (c) determine whether the demand is elastic or inelastic at $x = 3$. (Justify your answer.)

Answers

(1a) -8 ; (1b) $\frac{1}{2}$; (1c) 2 ; (1d) 3 ; (1e) 5 ; (1f) $-\infty$

(2a) $+\infty$; (2b) 1 ; (2c) -1 ; (2d) -3 ; (2e) $-\infty$; (2f) -1 ; (2g) 0 ; (2h) 2

(3a) $x = -3, x = 1$; (3b) $x = -2$; (4) discontinuous at $x = 5$; (5) $k = \pm 2$

(6a) Use $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ to find $f'(x) = \frac{-1}{2\sqrt{5-x}}$; (6b) $y = -\frac{1}{4}x + \frac{9}{4}$

(7a) $\frac{dy}{dx} = 5\sqrt{x^2+1} + 5x \frac{2x}{2\sqrt{x^2+1}}$; (7b) $\frac{dy}{dx} = 9x^{1/2} - \frac{5}{2}x^{-2} - \frac{e}{2}x^{-3/2}$

(7c) $\frac{dy}{dx} = \frac{3 \cos 3x(3 - 2 \cos x) - 2 \sin x \cdot \sin 3x}{(3 - 2 \cos x)^2}$; (7d) $\frac{dy}{dx} = 2 \sec^2(2x) + e^{\sec 3x} \cdot 3 \sec 3x \tan 3x$

(7e) $\frac{dy}{dx} = \frac{-\sin x}{\cos x} - \frac{1}{3} \frac{3x^2}{x^3+2} - 2 \frac{3}{3x-1}$; (7f) $\frac{dy}{dx} = \frac{4x^3-3}{(x^4-3x) \ln 7} + 3(2^{3x}) \ln 2$

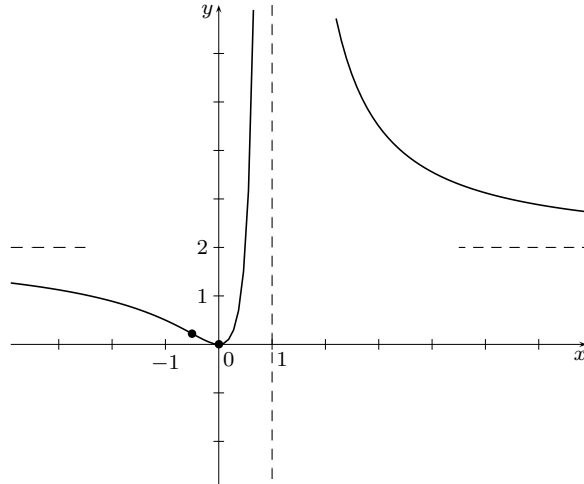
(7g) $\frac{dy}{dx} = \frac{2xy^3}{e^y - 3x^2y^2}$; (7h) $\frac{dy}{dx} = (3x^2+5)^{\sqrt{x}} \left[\frac{1}{2\sqrt{x}} \ln(3x^2+5) + \sqrt{x} \frac{6x}{3x^2+5} \right]$

(8) $f''(0) = 4$; (9) $x = -1, x = \frac{3}{2}$

(10) absolute maximum is 28 at $x = 5$; absolute minimum is -24 at $x = 3$

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(11)

 x and y intercept: $(0,0)$ vertical asymptote: $x = 1$ horizontal asymptote: $y = 2$ relative minimum: $(0,0)$ points of inflection: $(-\frac{1}{2}, \frac{2}{9})$ increasing: $0 < x < 1$ decreasing: $x < 0$ or $x > 1$ concave up: $-\frac{1}{2} < x < 1$ or $x > 1$ concave down: $x < -\frac{1}{2}$ (12) relative minimum at $(-2, -67)$; (13) the price is \$14,000 to maximize the revenue(14) the marginal cost is $C'(50) = -14700$ dollars/unit; $C'(50) \approx C(51) - C(50)$

(15) the dimensions are 25 by 50 meters to maximize the area.

(16a) $\eta = -\frac{x^2 + 7}{x^2}$; (16b) $\eta(3) = -\frac{16}{9} \approx -1.78$; (16c) demand is elastic