(Marks)

1. Find the following integrals without Integration Tables

(2) (a) $\int (x^{\pi} - \tan x + \pi^{x} + \cos \pi)\ dx$

(4) (b) $\int \frac{x^3 + x^2 + 1}{x^2 + x - 2}\ dx$

(4) (c) $\int_1^e \frac{(3 + \ln x)^3}{x}\ dx$

(4) (d) $\int \frac{1}{\sqrt{x} + 4}\ dx$

(4) (e) $\int x^2 \sin x\ dx$

(6) (f) $\int \frac{-x^2 + 7x + 12}{x^3 + 4x^2}\ dx$

2. Find the following integrals by using the integration tables provided.
In each case, state the formula number and justify its use.

(4) (a) $\int \frac{1}{\sqrt{x^2 + 4x - 5}}\ dx$

(4) (b) $\int \frac{x}{9 - (x^2 + 2)^2}\ dx$

(4) 3. Sketch of the region bounded by the graphs of $f(x) = 4x - x^2 + 8$ and $g(x) = x^2 - 2x$ and find the area of that region.

(5) 4. Given: Demand, $p = 48 - x^2$ and Supply, $p = x^2 + 16x + 8$.

(a) Find the Equilibrium Point.

(b) Graph the demand and supply functions. Clearly indicate equilibrium point and the regions corresponding to Consumer's Surplus and Producer's Surplus.

(c) Find the Producer's Surplus. Round your answer to 2 decimal places.

(4) 5. Use Trapezoidal Rule with $n = 6$ subintervals to approximate $\int_2^5 \frac{x}{\sqrt{x - 1}}\ dx$.
Round your answer to 4 decimal places.

6. Solve the following differential equations for y:

(3) (a) $y'' = 4x^3 + e^{2x}$ when $y'(0) = 1$ and $y(0) = 4$

(3) (b) $\frac{dy}{dx} = \ln x$ when $y(1) = 5$

7. Provide the general solution to each of the following differential equation :

(3) (a) $x\frac{dy}{dx} - \frac{x - 4}{4y^2} = 0$ where $x > 0$

(3) (b) $(x^2 + 1)\, y' = x\, y$ where $y > 0$

(6) 8. A small community in northern Quebec is affected by the H1N1 virus. A vaccine is not immediately available to this community and will not be available for some time. The number of persons affected is growing at a rate proportional to the number affected at any time t. Initially, there were 3 persons affected and 7 after 2 weeks.

(a) How many persons will be affected at the end of 5 weeks?

(b) How long will it take for 30 persons to be affected?

(Marks)

9. Find each limit, if it exists.

(3) (a) $\lim_{x\to 0} \dfrac{\sin x - x}{x^3}$

(3) (b) $\lim_{x\to 3^-} \dfrac{\sqrt{9-x^2}}{x-3}$

10. Determine whether the following improper integral is convergent or divergent.
Evaluate each convergent integral.

(4) (a) $\displaystyle\int_{-\infty}^{1} x\, e^{x^2}\, dx$

(4) (b) $\displaystyle\int_{4}^{5} \frac{1}{\sqrt[3]{x-5}}\, dx$

(6) 11. Given the sequence : $9\,,\ \dfrac{13}{2}\,,\ \dfrac{17}{3}\,,\ \dfrac{21}{4}\,,\ \ldots$

(a) Write the next 4 terms

(b) Find a formula for the n^{th} term, a_n

(c) Does the given sequence converge or diverge? Justify.

(2) 12. A sequence $\{a_n\}$ is given by $a_1 = 1\,,\ a_2 = 3$ and $a_{n+1} = 2\,a_n - a_{n-1}$ for $n \geq 2\,.$
List the first 6 terms and give another formula for a_n

(2) 13. Find a formula for the n^{th} term, a_n for the sequence $\dfrac{1}{5}\,,\ -\dfrac{2}{25}\,,\ \dfrac{6}{125}\,,\ -\dfrac{24}{625}\,,\ \ldots$

14. Determine the convergence or divergence of each sequence. If the sequence converges, find the limit.

(2) (a) $a_n = (-1)^n \dfrac{5n+1}{3n-3}$

(2) (b) $a_n = \dfrac{\ln\,(n+e^n)}{n}$

(2) 15. A deposit of \$2000 is made at 3% interest compounded monthly. Find the balance after 3 years, given that the monthly compound interest sequence is given by $A_n = P\left(1+\dfrac{r}{12}\right)^n$.

16. Determine with justification if each of the following series is convergent or divergent.
If the series is convergent, find its sum:

(2) (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n+3}{\sqrt{n^2+1}}$

(3) (b) $\displaystyle\sum_{n=1}^{\infty} \frac{3^{n+1}}{5^n}$

(2) 17. A deposit of \$40 is made at the beginning of every month for four years into a savings account that collects 1.4% a year compounded monthly. Find the balance A at the end of four years.
Use $A = \sum A_n$ where $A_n = 40\left(1+\dfrac{0.014}{12}\right)^n$