Part 1 - 1 mark each

True/False: Answer true (T) if the statement is always true. Answer false (F) otherwise

1. For the sample data set $x_1 , x_2 , \ldots x_n ; \sum_{i=1}^{n} x_i^2 = (n-1)\overline{x}^2 + ns^2$
2. If $A$ is any event on $S$ with $P(A) = P(A')$ , then $P(A) = 0.50$
3. For any random variable $X$ , $V(aX + b) = (a + b)^2 V(X)$
4. The sample mean $\overline{x}$ is a normally distributed random variable
5. If $X$ is a continuous r.v. with c.d.f. $F(x)$ , then for any $b$ , $P(x > b) = 1 - F(b)$
6. If $X$ is a continuous r.v. defined on $[-5, 7]$ , then $V(X) = 12$
7. The p.m.f. of the hypergeometric distribution has exactly 2 parameters
8. The IQR of the std. normal r.v. z is approximately 1.34
9. All else equal, the width of a C.I. for $\mu$ gets narrower as $\sigma$ gets larger
10. $\beta = P(\text{rejecting } H_0 \text{ when } H_0 \text{ is true})$

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Part 2 - 3 marks each

1. According to chebychev's theorem, at least 50% of a sample lies between 50 and 70. Find the mean and std deviation of this sample.
2. The mean of a sample is 40.0 . The "new" mean becomes 39.5 when the number "36" is added to this sample. What is the sample size (n) before this number is added.
3. An experiment consists of rolling a balanced die once. If an even number occurs, the experiment ends. If an odd number occurs, a fair coin is tossed twice and the experiment ends
   (a) State the sample space S for this experiment.
   (b) Find the probability of getting 2 heads
4. For a Poisson r.v. $x$ with $\lambda = 4$ , find $P(|x - \mu| < \sigma)$
5. $X$ is a continuous r.v. with p.d.f. $f(x) = \begin{cases} \frac{k}{x^2} & 2 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$

   Find $k$ and use it to compute $E(x)$
6. If $X \sim b(n, p)$ , show that $E(X^2) = np(np + 1 - p)$
7. A $(1-\alpha)$ confidence interval for $\mu$ based on a sample of $n = 16$ is $50 < \mu < 60$ . If $\sigma = 8$ , find $(1-\alpha)$.
8. $\hat{\mu}_1 = \frac{2x_1+3x_2}{5}$ and $\hat{\mu}_2 = \frac{5x_1-2x_2}{3}$ are estimators of $\mu$ , the mean of a population with variance $\sigma^2$
   (a) Show that both estimators are unbiased
   (b) Which has minimum variance?

9. If $\sigma_1 = \sigma_2 = \sqrt{8}$, find the common sample size required to estimate $\mu_1 - \mu_2$ witha margin of error of 1.5 units, 18 times in 20.

10. For the hypothesis set, $H_o : \frac{\sigma_1^2}{\sigma_2^2} = 1$ vs $H_a : \frac{\sigma_1^2}{\sigma_2^2} > 1$, if $\alpha = 0.1$, $n_1 = 13$, $n_2 = 7$ and $s_2^2 = 10$, find the smallest value of $s_1$ for which $H_0$ can be rejected.

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Part 3 - 10 marks each

1. One half of 1% of a certain community has H1N1 virus. A test has been developed to identify the presence of this virus. This test has a false positive rate of 0.1% and a false negative rate of 0.2%. If the test is administered to a randomly selected person from this community and the presence of the virus is indicated − what is the probability that this person is infected?

2. The p.m.f. of a discrete (uniform) r.v. $x$ is given by $P(x) = \begin{cases} \frac{1}{n} & x = 1, 2, 3, 4 \ldots n \\ 0 & \text{otherwise} \end{cases}$

   If $n = 11$, compute $P(|x - \mu| < \sigma)$

3. A drug manufacturer produces 250-milligram capsules of a new antibiotic. A random sample of 10 such capsules is selected and the amount of antibiotic in each capsule is determined. The results are: 252,246,242,250,255,258,250,252,250,258. Use this sample to:
   (a) find point estimates of the population mean and std. deviation.
   (b) find the 95% upper confidence bound for $\mu$
   (c) find the 95% confidence interval estimate for $\sigma$ .

4. Suppose a physician wants to estimate the proportion of cancer patients who survive more than 5 years after diagnosis.
   (a) How large a sample should she select if she wants to estimate this proportion to within 1%, 19 times in 20?
   (b) If a sample of size $n$ found in (a) resulted in 1500 patients who survive for more than 5 years after diagnosis − would you consider this as sufficient evidence that more than 15% (of the population of cancer patients) survive for longer than 5 years?
   Test by the $p-$value method $(\alpha = 0.02)$ .
   (c) If the true percentage of post 5 years survivors is actually 20%, find the probability of type 2 error.

5. A sociologist is studying the effects of a certain motion picture film on the attitudes of white men towards black men. Twelve white men were randomly selected and asked to fill out a questionnaire before and after viewing the film. The scores are given below:

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Before: | 11 | 13 | 19 | 13 | 9 | 8 | 14 | 13 | 18 | 21 | 7 | 12 |
| After: | 6 | 9 | 12 | 16 | 4 | 5 | 10 | 14 | 13 | 17 | 7 | 11 |

   Assuming normal scores:
   (a) Find a 90% confidence interval for the mean shift in score that takes place after the film is viewed.
   (b) At the 2% level, has there been a significant mean shift in attitude?

6. It is believed that smoking boosts death risk for diabetics. A scientist investigated the smoking rates for male and female diabetics and obtained the following data:

| Gender | $n$ | # who smoke |
|---|---|---|
| male | 400 | 172 |
| female | 400 | 136 |

(a) Find a 90% upper confidence bound for the difference in proportion of male and female smokers

(b) Test the research hypothesis that the smoking rate for males is higher than that of females. Complete the test using the $p-$value method ($\alpha = 0.05$)

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**Answers**

Part 1: (1) F ; (2) T ; (3) F ; (4) F ; (5) T ; (6) T ; (7) F ; (8) T ; (9) F ; (10) F

Part 2: (1) $S = 5\sqrt{2}$ ; (2) n=7

(3 a) $S = \{2, 4, 6, 1HH, 1HT, 1TH, 1TT, 3HH, \ldots 3TT, 5HH, \ldots 5TT\}$ ; (3 b) $\frac{1}{8}$

(4) $P(|x-4| < 2) = P(2 < x < 6) = P(x = 3, 4, 5) = e^{-4}\left(\frac{4^3}{3!} + \frac{4^4}{4!} + \frac{4^5}{5!}\right) \approx 0.547$

(5) $k = 4$ , $E(x) = 4(\ln 4 - \ln 2) = 4 \ln 2$

(6) $E(X^2) = V(X) - (E(X))^2 = np(1-p) + n^2p^2 = np(np + 1 - p)$

(7) $1 - \alpha = P(-2.5 < z < 2.5) = 0.9876$

(8 a) $E(\hat{\mu_1}) = \mu = E(\hat{\mu_2})$ ; (8 b) $V(\hat{\mu_1}) = \frac{13}{25}\sigma^2$ , $V(\hat{\mu_2}) = \frac{29}{9}\sigma^2$ , $V(\hat{\mu_1}) < V(\hat{\mu_2})$

(9) $n = \left(\frac{1.645}{1.5}\right)^2 (8+8) \approx 20$

(10) $s_1 > \sqrt{29}$

Part 3: (1) $p = 0.834$ ; (2) $\mu = 6$ , $\sigma = \sqrt{10}$ , $P(2.83 < x < 9.16) = P(x = 3, 4, \ldots, 8, 9) = \frac{7}{11}$

(3 a) $\overline{x} = 251.3$ , $s = 4.99$ ; (3 b) $\mu < 254.19$ ; (3 c) $3.432 < \sigma < 9.11$

(4 a) $n = \left(\frac{1.96}{0.01}\right)^2 \frac{1}{4} = 9604$

(4 b) $H_0 :\ p = 0.15$ , $H_a :\ p > 0.15$ , $z^* \approx 1.7 < z(\alpha) = 2.05$ ; do not reject $H_0$

(4 c) $\beta = \Phi\left[\frac{-0.05-0.00713}{0.004082}\right] \approx 0$

(5 a) $d = 2.833$ , $s_d = 2.949$ ; $1.304 < \mu_d < 4.362$

(5 b) $H_0 :\ \mu_d = 0$ , $H_a :\ \mu_d \neq 0$ , $t^* = 3.328 > t(11.01) = 2.718$ ; reject $H_0$

(6 a) $\hat{p_1} = 0.43$ , $\hat{p_2} = 0.34$ ; upper confidence bound $= 0.1339$

(6 b) $H_0 :\ p_1 - p_2 = 0$ , $H_a :\ p_1 - p_2 > 0$ ; $z^* = 2.616$

$p-$value $= P(z > 2.616) = 1 - 0.9956 = 0.0044 < \alpha = 0.05 \Rightarrow$ reject $H_0$