

[Marks]

- [6] 1. True or False?
- Every solution to $\frac{1}{x} < x$ must satisfy $x > 1$.
 - The expression $7^{\log_{49}(2)}$ is equal to a rational number.
 - The inequality $2 \leq |3 \cos(\theta) - 5| \leq 8$ is true for all θ .
 - The equation $\ln(x^2 - 1) = \ln(x + 1) + \ln(x - 1)$ is true for all real numbers x .
 - The polynomial $x^{2011} - 3x^{1001} + 1$ has at least one real zero.
 - The polynomial $x^{2011} - 3x^{1001} + 1$ has at least one rational zero.
2. Solve each of the following for x . Use interval notation where appropriate.
- [2] a. $\frac{x^2}{x^2 - 9} \geq 0$
- [2] b. $|3 - x| = |5x - 14|$
- [2] c. $|1 - 2x| < 7$
- [3] 3. A parabola has vertex at $(-5, -12)$ and y -intercept $(0, 63)$. Find the x -intercepts.
- [3] 4. Find the center and radius of the circle given by $x^2 + y^2 - 10x + 8y = 8$.
- [3] 5. Consider the line $y = \frac{4}{3}x + 7$. There are two points on the line that have a distance 10 away from the y -intercept. Find these two points. (Be sure to give two coordinates for each point.)
- [3] 6. The polynomial $P(x) = 3x^3 - 26x^2 + 51x + 20$ has three real zeros. Given that $x = 5$ is one of them, find the other two.
- [3] 7. Find the quotient and remainder when $8x^5 - 2x^4 + 3x^2 - 20x + 5$ is divided by $2x^3 - 3x$.
8. Find the domains of each of the following functions. Express your answer in interval notation.
- [2] a. $f(x) = \sqrt{\frac{x^2 - 1}{x^2 - 4}}$
- [2] b. $f(x) = \frac{\sqrt{x^2 - 1}}{x^2 - 4}$

[Marks]

[6] 9. For the rational function $f(x) = \frac{-3(x+2)(x-1)}{(x+1)^2}$.

- Find the domain.
- Find all x and y intercepts.
- Find all vertical and horizontal asymptotes.
- Find the coordinates of any hole(s) the graph of $f(x)$ might have.
- Sketch the graph on the given axes.

[7] 10. For the rational function $f(x) = \frac{-5x^2 + 10x}{x^3 - 4x^2 - 21x}$.

- Find the domain.
- Find all x and y intercepts.
- Find all vertical and horizontal asymptotes.
- Find the coordinates of any hole(s) the graph of $f(x)$ might have.
- Sketch the graph on the given axes.

[4] 11. Let $f(x) = 4 - \sqrt{x-2}$.

- Find the domain and range of $f(x)$.
- Find an expression for $f^{-1}(x)$.
- Find the domain and range of $f^{-1}(x)$.
- Sketch f^{-1} on the given axes.

[3] 12. Sketch a graph of the function $y = 4 - 2^{x+3}$ on the given axes. Clearly indicate all intercepts and asymptotes.

[3] 13. Given the function $f(x) = 1 - (x-2)^2$, sketch a graph of $g(x) = \frac{1}{f(x)}$ on the given axes. Clearly indicate all intercepts and asymptotes.

14. Let $f(x) = \frac{3x+1}{x-2}$ and $g(x) = x^2 + x$.

[2] a. Find the domain of $(f \circ g)(x)$.

[2] b. Find $f^{-1}(x)$.

[3] 15. Find A , B , and C given: $\ln\left(\frac{e^7 \sqrt[3]{x}}{y^5}\right) = A + B \ln x + C \ln y$.

[2] 16. Simplify: $\ln\left(\frac{e^a e^{-2b}}{e^{3c} e^{-4d}}\right)$.

[Marks]

17. Solve each of the following for x . Give exact values.

[2] a. $2 \ln x + \ln 2 = \ln(1 - x)$

[2] b. $\log_2(x) + \log_2(x - 6) = 4$.

[2] c. $6^{(x+1)} = 3 \cdot 2^x$

[1] 18. Convert -404° to radians. Give an exact value in simplest form.[3] 19. Given the function $f(x) = -4 \sin(x + \pi/4)$,

a. state the amplitude and period.

b. sketch the graph on the given axes.

[3] 20. Given the function $f(x) = 3 + \cos(2x - \pi)$,

a. state the amplitude and period.

b. sketch the graph on the given axes.

[3] 21. If $\sec \theta = 5$ and $\tan \theta < 0$, find $\sin \theta$.[3] 22. If $\cos \theta = 5/9$ and $\sin \theta > 0$, find $\cos(\theta + \pi/6)$.23. Find all x in $[0, 2\pi]$ that satisfy the following equations. Give exact values in radians.

[3] a. $\tan^2 x + \tan x = 0$

[3] b. $\cos(2x) = 0$

[2] c. $\ln(\sin x) = 0$

[2] 24. Verify the identity assuming the angle θ is acute: $\cos \theta = \frac{1}{\csc \theta \sqrt{\sec^2 \theta - 1}}$.[2] 25. Verify the identity: $\frac{\cos^2 x}{1 + \sin x} = 1 - \sin x$

CALCULATOR PROBLEMS. Round your answers to three decimal places.

[2] 26. Solve: $(3/2)^t = 100$.27. A triangle has angle measures α, β , and γ , opposite sides of lengths a, b , and c , respectively.[2] a. If $\alpha = 100^\circ$, $b = 10$ and $c = 7$, find a .[2] b. If $\alpha = 100^\circ$, $a = 20$ and $b = 9$, find γ .

[Marks]

ANSWERS. See bottom for graphs.

- 1a. F 1b. F 1c. T 1d. F 1e. T 1f. F
 2a. $(-\infty, -3) \cup (3, \infty)$ 2b. $x = 17/6, x = 11/4$ 2c. $(-3, 4)$ 3. $(-3, 0)$ and $(-7, 0)$
 4. Center: $(5, -4)$ and radius: $r = 7$ 5. $(6, 15)$ and $(-6, -1)$ 6. $x = -1/3, x = 4$
 7. Q: $4x^2 - x + 6$, R: $-2x + 5$ 8a. $(-\infty, -2) \cup [-1, 1] \cup (2, \infty)$ 8b. $(-\infty, -2) \cup (-2, -1] \cup [1, 2) \cup (2, \infty)$
 9a. $\mathbb{R} \setminus \{-1\}$ 9b. y -int: $(0, 6)$, x -ints: $(-2, 0)$ and $(1, 0)$ 9c. VA: $x = -1$, HA: $y = -3$ 9d. No holes.
 10a. $\mathbb{R} \setminus \{-3, 0, 7\}$ 10b. y -int: none, x -int: $(2, 0)$ 10c. VA: $x = -3$ and $x = 7$, HA: $y = 0$ 10d. $(0, -10/21)$
 11a. Domain: $[2, \infty)$, Range: $(-\infty, 4]$ 11b. $f^{-1}(x) = (x - 4)^2 + 2$ 11c. Domain: $(-\infty, 4]$, Range: $[2, \infty)$
 14a. $\mathbb{R} \setminus \{-2, 1\}$ 14b. $f^{-1}(x) = \frac{2x+1}{x-3}$ 15. $A = 7, B = 1/3, C = -5$ 16. $a - 2b - 3c + 4d$ 17a. $x = 1/2$ 17b. $x = 8$ 17c. $x = -\frac{\ln 2}{\ln 3}$ 18. $-101\pi/45$ 19. Amp=4, Per= 2π 20. Amp=1, Per= π
 21. $-2\sqrt{6}/5$ 22. $\frac{5\sqrt{3}-2\sqrt{14}}{18}$ 23a. $x = 0, 3\pi/4, \pi, 7\pi/4, 2\pi$ 23b. $x = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$
 23c. $x = \pi/2$ 26. $t \approx 11.358$ 27a. $a \approx 13.165$ 27b. $\gamma \approx 53.694^\circ$

