Any function appearing under a sign of integration is assumed continuous throughout the interval of integration.

Basic properties of the definite integral

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx \qquad \text{(Interval additivity)}$$

If $a < b$ and $f(x) < g(x)$ on (a, b) then $\displaystyle\int_a^b f(x)\,dx < \int_a^b g(x)\,dx.$ (Monotonicity)

$$\int_a^b \alpha\,dx = \alpha(b-a) \qquad \text{(Integral of a constant)}$$

If $a < b$ and g does not change sign on (a, b), then

$$\int_a^b f(x)g(x)\,dx = f(c)\int_a^b g(x)\,dx \text{ for some } c \in (a, b). \qquad \text{(Mean Value Theorem for integrals)}$$

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x) \qquad \text{(First Fundamental Theorem of Calculus)}$$

$$\int_a^b f'(x)\,dx = f(x)\Big|_a^b = f(b) - f(a) \qquad \text{(Second Fundamental Theorem of Calculus)}$$

$$\int_a^b \big(\alpha f(x) + \beta g(x)\big)\,dx = \alpha\int_a^b f(x)\,dx + \beta\int_a^b g(x)\,dx \qquad \text{(Linearity)}$$

$$\int_a^b f\big(g(x)\big)g'(x)\,dx = \int_{g(a)}^{g(b)} f(t)\,dt \qquad \text{(Change of variables)}$$

$$\int_a^b f(x)g'(x)\,dx = f(x)g(x)\Big|_a^b - \int_a^b f'(x)g(x)\,dx \qquad \text{(Partial integration)}$$

To linearity, change of variables and partial integration correspond analogous properties of indefinite integrals.

In the indefinite integrals below the constant of integration is suppressed: $\int f = F$ simply means that $F' = f$.

Basic indefinite integrals of power and exponential functions

$$\int t^\alpha\,dt = \frac{t^{\alpha+1}}{\alpha+1} \quad \text{for} \quad \alpha \neq -1, \quad \text{and} \quad \int \frac{dt}{t} = \log|t|. \qquad \text{(Powers)}$$

$$\int e^t\,dt = e^t, \quad \text{and} \quad \int a^t\,dt = \frac{a^t}{\log a} \quad \text{for } a > 0 \quad \text{and} \quad a \neq 1. \qquad \text{(Exponentials)}$$

Basic indefinite trigonometric integrals

$$\int \sin(t)\,dt = -\cos(t) \quad \text{and} \quad \int \cos(t)\,dt = \sin(t).$$

$$\int \tan(t)\,dt = -\log|\cos(t)| \quad \text{and} \quad \int \cot(t)\,dt = \log|\sin(t)|.$$

$$\int \sec(t)\,dt = \log|\sec(t) + \tan(t)| \quad \text{and} \quad \int \csc(t)\,dt = -\log|\csc(t) + \cot(t)|.$$

$$\int \sec^2(t)\,dt = \tan(t) \quad \text{and} \quad \int \csc^2(t)\,dt = -\cot(t).$$

$$\int \sec(t)\tan(t)\,dt = \sec(t) \quad \text{and} \quad \int \csc(t)\cot(t)\,dt = -\csc(t).$$

Basic indefinite integrals of rational functions

$$\int \frac{dt}{t^2 + a^2} = \frac{1}{a}\arctan\left(\frac{t}{a}\right) \quad \text{and} \quad \int \frac{dt}{t^2 - a^2} = \frac{1}{2a}\log\left|\frac{t-a}{t+a}\right| \quad \text{for } a > 0.$$

Basic indefinite integrals of algebraic functions

$$\int \frac{dt}{\sqrt{a^2 - t^2}} = \arcsin\left(\frac{t}{a}\right) \quad \text{and} \quad \int \frac{dt}{\sqrt{t^2 \pm a^2}} = \log\left|t + \sqrt{t^2 \pm a^2}\,\right| \quad \text{for } a > 0.$$

Any function appearing under a sign of integration is assumed continuous throughout the interval of integration.

Basic properties of the definite integral

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx \qquad \text{(Interval additivity)}$$

If $a < b$ and $f(x) < g(x)$ on (a, b) then $\displaystyle\int_a^b f(x)\,dx < \int_a^b g(x)\,dx.$ (Monotonicity)

$$\int_a^b \alpha\,dx = \alpha(b-a) \qquad \text{(Integral of a constant)}$$

If $a < b$ and g does not change sign on (a, b), then

$$\int_a^b f(x)g(x)\,dx = f(c)\int_a^b g(x)\,dx \text{ for some } c \in (a, b). \qquad \text{(Mean Value Theorem for integrals)}$$

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x) \qquad \text{(First Fundamental Theorem of Calculus)}$$

$$\int_a^b f'(x)\,dx = f(x)\Big|_a^b = f(b) - f(a) \qquad \text{(Second Fundamental Theorem of Calculus)}$$

$$\int_a^b \big(\alpha f(x) + \beta g(x)\big)\,dx = \alpha\int_a^b f(x)\,dx + \beta\int_a^b g(x)\,dx \qquad \text{(Linearity)}$$

$$\int_a^b f\big(g(x)\big)g'(x)\,dx = \int_{g(a)}^{g(b)} f(t)\,dt \qquad \text{(Change of variables)}$$

$$\int_a^b f(x)g'(x)\,dx = f(x)g(x)\Big|_a^b - \int_a^b f'(x)g(x)\,dx \qquad \text{(Partial integration)}$$

To linearity, change of variables and partial integration correspond analogous properties of indefinite integrals.

In the indefinite integrals below the constant of integration is suppressed: $\int f = F$ simply means that $F' = f$.

Basic indefinite integrals of power and exponential functions

$$\int t^\alpha\,dt = \frac{t^{\alpha+1}}{\alpha+1} \quad \text{for} \quad \alpha \neq -1, \quad \text{and} \quad \int \frac{dt}{t} = \log|t|. \qquad \text{(Powers)}$$

$$\int e^t\,dt = e^t, \quad \text{and} \quad \int a^t\,dt = \frac{a^t}{\log a} \quad \text{for } a > 0 \quad \text{and} \quad a \neq 1. \qquad \text{(Exponentials)}$$

Basic indefinite trigonometric integrals

$$\int \sin(t)\,dt = -\cos(t) \quad \text{and} \quad \int \cos(t)\,dt = \sin(t).$$

$$\int \tan(t)\,dt = -\log|\cos(t)| \quad \text{and} \quad \int \cot(t)\,dt = \log|\sin(t)|.$$

$$\int \sec(t)\,dt = \log|\sec(t) + \tan(t)| \quad \text{and} \quad \int \csc(t)\,dt = -\log|\csc(t) + \cot(t)|.$$

$$\int \sec^2(t)\,dt = \tan(t) \quad \text{and} \quad \int \csc^2(t)\,dt = -\cot(t).$$

$$\int \sec(t)\tan(t)\,dt = \sec(t) \quad \text{and} \quad \int \csc(t)\cot(t)\,dt = -\csc(t).$$

Basic indefinite integrals of rational functions

$$\int \frac{dt}{t^2 + a^2} = \frac{1}{a}\arctan\left(\frac{t}{a}\right) \quad \text{and} \quad \int \frac{dt}{t^2 - a^2} = \frac{1}{2a}\log\left|\frac{t-a}{t+a}\right| \quad \text{for } a > 0.$$

Basic indefinite integrals of algebraic functions

$$\int \frac{dt}{\sqrt{a^2 - t^2}} = \arcsin\left(\frac{t}{a}\right) \quad \text{and} \quad \int \frac{dt}{\sqrt{t^2 \pm a^2}} = \log\left|t + \sqrt{t^2 \pm a^2}\,\right| \quad \text{for } a > 0.$$